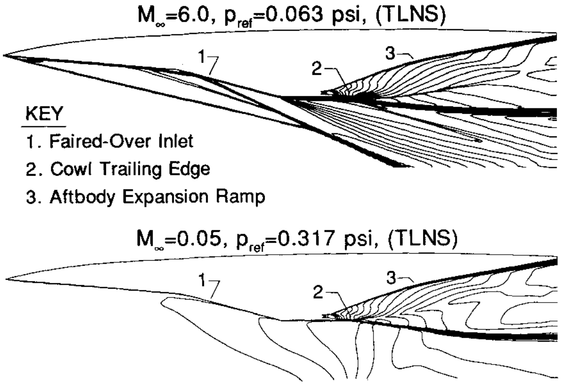

Fig. 3 Flowfield comparison of pseudostatic- and dynamic-powered simulation for the powered hypersonic vehicle model with consistent reference pressure near the plume, Mach number contours.

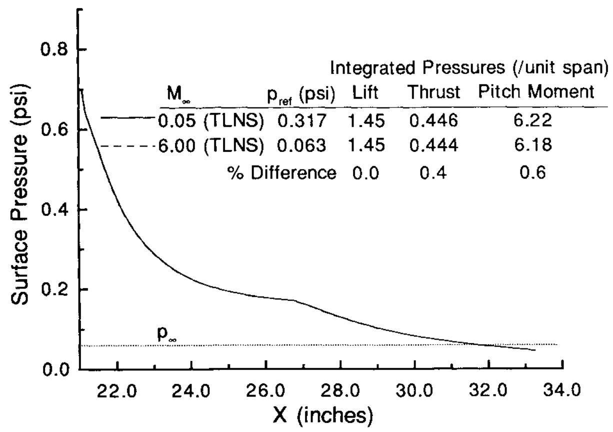

Fig. 4 Afterbody surface pressure comparison of pseudostatic- and dynamic-powered simulation for the powered hypersonic vehicle model with consistent reference pressure near the plume.

in the local flow region between the external cowl-trailing-edge shock and the plume near the cowl trailing edge. The value for this reference static pressure was five times the freestream static pressure, or 0.317 psia. A new pseudostatic TLNS solution was computed, this time at $M_\infty = 0.05$, which was possible because the solution became more stable using the higher reference pressure.

In addition to the question of appropriate choice of reference pressure, the difference in the aftbody surface pressures near the cowl trailing edge (Fig. 2) may have been due to the numerical modeling instead of the physical conditions, since one result was a PNS solution and the other was a TLNS solution. Therefore, a new hypersonic solution was also computed at the same conditions as before, namely, $M_\infty = 6.0$ and $Re_\infty = 2.0 \times 10^6$/ft, and $p_\infty = 0.063$ psia, but using the TLNS equations instead of the PNS equations.

The new solutions converged with a reduction in the global residual of four orders of magnitude in 868 and 1201 iterations for the pseudostatic and hypersonic cases, respectively. A comparison of flowfields shown in Fig. 3 reveals that the plume location for the pseudostatic solution is very close to that for the hypersonic case. The location of the plume in the hypersonic solution did not change by applying the TLNS equations to the problem. Examination of the aftbody surface pressures for the two TLNS solutions (Fig. 4) shows nearly identical pressure distributions and integrated force and moment components. Furthermore, the force and moment components from the pseudostatic solution shown in Fig. 2 are also nearly identical to those from the pseudostatic solution shown in Fig. 4, implying that the 3% difference seen in Fig. 2 is not caused by the use of a different reference pressure, but is actually a numerical phenomenon associated with the equation set being solved.

Conclusions

Two-dimensional CFD analyses have been presented related to the ground testing of hypersonic, air-breathing models that feature scramjet exhaust flow simulation. CFD analysis shows that it is possible to test aftbody powered hypersonic airbreather configurations in a static, pumped-down environment to obtain aftbody aerodynamic performance data. However, the analysis shows that a tunnel static pressure must be used in order to provide a comparable reference pressure that occurs at the location where the plume propagates off the cowl trailing edge in the hypersonic flow, instead of simply using the freestream static pressure that would be achieved in the hypersonic flowfield.

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Formulation of Design Envelope Criterion in Terms of Deterministic Spectral Procedure

J. G. Jones*

Defence Research Agency, RAE Farnborough, Hampshire, GU14 6TD, England, United Kingdom

Introduction

MANDATORY aircraft limit-load requirements for flight in continuous turbulence are generally met by using power-spectral procedures to compute the loads. Two methods are in general use: 1) design envelope approach, and 2) mission analysis. In the design envelope approach, a response factor $\bar{A}$ is calculated [Eq. (8)] and multiplied by a specified gust intensity U_σ to obtain the design load for a series of points throughout the design envelope. In mission analysis, mission profiles are analyzed in order to obtain probabilities of exceeding various load levels and a design probability is specified from which design loads may be found.

In this note we consider the design envelope approach and demonstrate that it can be reformulated in a manner which makes no distinction between linear and nonlinear response,

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*Senior Principal Scientific Officer. Member AIAA.

thus providing a rational basis for its extension to the nonlinear aircraft. The reformulation depends upon the demonstrated result[1,2] that, whereas power-spectral procedures for evaluating the stochastic response of linear systems are generally implemented by means of frequency-plane calculations, an alternative—but exact equivalent—time-plane method exists in the form of a worst-case analysis in which the maximum response to a class of deterministic inputs subject to a prescribed constraint is found. This is shown to lead naturally to a deterministic spectral procedure (DSP) for implementing design envelope requirements and being equally applicable to linear and nonlinear aircraft. This note is a condensation of a previously published memorandum[3] with limited circulation.

Equivalent Deterministic Analysis

The starting point is a result concerning the maximum response of a linear system to a deterministic input subject to a prescribed constraint.[4] The system frequency-response function will be denoted by $H_y(i\omega)$, and a deterministic input by $x(t)$ or its associated Fourier transform $X(\omega)$. The prescribed constraint on the input takes the form

$$\pi \int_0^\infty |X(\omega)|^2 G(\omega)\, \mathrm{d}\omega \leq 1 \tag{1}$$

where $G(\omega)$ is an arbitrary positive function $G(\omega) \geq 0$. It can be demonstrated that the weak inequality in Eq. (1) may be replaced by an equality without altering any of the subsequent results. However, for the present purpose of extending the method to apply to nonlinear systems, the constraint as expressed in Eq. (1) has its advantages.

By application of the Schwartz inequality, Papoulis[4] showed that the maximum amplitude $\max_x |y(t)|$ of system response $y(t)$ when the deterministic input $x(t)$ is varied over the class of functions subject to the constraint of Eq. (1), is given by

$$[\max_x |y(t)|]^2 = \frac{1}{\pi} \int_0^\infty \frac{|H_y(i\omega)|^2}{G(\omega)}\, \mathrm{d}\omega \tag{2}$$

This result may be used[1,2] to provide a bridge between a system analysis based on a deterministic worst-case search or standard power-spectral methods to determine the dynamic response of linear systems to stochastic inputs. The usual basis[5] for the latter approach is the equation

$$\sigma_y^2 = \int_0^\infty |H_y(i\omega)|^2 \phi(\omega)\, \mathrm{d}\omega \tag{3}$$

for the variance of the response variable $y(t)$, having frequency-response function $H_y(i\omega)$ due to a stationary stochastic input with power spectrum $\phi(\omega)$. That is, σ_y^2 is evaluated as an integral in the frequency plane. However, by relating $G(\omega)$ to the inverse of the power spectral density of the input

$$G(\omega) = [2\pi\phi(\omega)]^{-1} \tag{4}$$

it follows from Eqs. (2) and (3) that

$$\sigma_y^2 = \frac{[\max_x |y(t)|]^2}{2} \tag{5}$$

Therefore, the variance of the stochastic response may be obtained by evaluating the maximum response resulting from a deterministic worst-case analysis. In this analysis, the maximum response $\max_x |y(t)|$ is found with respect to a class of deterministic inputs $x(t)$ subject to the constraint

$$\frac{1}{2} \int_0^\infty \frac{|X(\omega)|^2}{\phi(\omega)}\, \mathrm{d}\omega \leq 1 \tag{6}$$

[from Eqs. (1) and (4)].

Deterministic Spectral Procedure

In the standard design envelope approach to limit loads,[6,7] which assumes a linear dynamic model for the aircraft, a design load y_d is calculated using the equation

$$y_d = \bar{A} U_\sigma \tag{7}$$

where U_σ is a turbulence intensity which is prescribed in the requirements[6,7] and depends on altitude and aircraft speed and $\bar{A}$ is an aircraft-dependent dynamic response factor calculated using PSD theory as the ratio of standard deviations of output and input:

$$\bar{A} = \frac{\sigma_y}{\sigma(\text{gust})} \tag{8}$$

In Eq. (8) σ_y is calculated in the frequency plane, using Eq. (3).

As pointed out explicitly in Ref. 8, Eqs. (7) and (8) combined with Eqs. (5) and (6) lead to the result

$$y_d = \max_u |y(t)| \tag{9}$$

where the maximum aircraft response is evaluated with respect to a deterministic family of gust inputs $u(t)$, subject to the constraint

$$\|u(t)\| \leq U_\sigma \tag{10}$$

and the "norm" of $u(t)$ is defined by the equation

$$\|u(t)\|^2 = \int_0^\infty \frac{|U(\omega)|^2}{\phi(\omega)}\, \mathrm{d}\omega \tag{11}$$

($U(\omega)$ is the Fourier transform of $u(t)$). Equations (9–11) form the basis of the proposed DSP.

The term $|U(\omega)|^2$, appearing in Eq. (11), may be interpreted as the contribution from frequency ω to the energy of the gust input $u(t)$. The quantity $\|u(t)\|^2$, which is subjected to a bound by Eq. (10), is thus a modified form of energy in which the frequency components are weighted by the inverse of the power spectrum.

Whereas Eq. (7) (the usual basis for design envelope calculations) is only applicable as it stands to linear aircraft dynamic models, the new procedure based on Eqs. (9–11), although derived specifically for the linear problem, contains no reference to linearity and is applicable equally when nonlinearities are present. The inequality [Eq. (10)] is in the spirit of a design envelope, in that a requirement formulated in these terms would extend the envelope comprising altitudes and speeds to encompass a specific family of gusts $u(t)$, namely those satisfying Eq. (10), to which the aircraft must be exposed (without exceeding its design load) at each altitude and speed condition.

Implementation Issues

The calculation of y_d on the basis of Eqs. (9–11) requires that a sequence of deterministic samples $u(t)$ be generated and that the maximum response $\max_u |y(t)|$ be found subject to the constraint imposed by Eq. (10). There are many ways of solving this problem of constrained optimization, from which a user should be free to choose. Here we simply make some general points about implementation, underline some possible pitfalls, and point to means of avoiding them.

To implement the search for the sample $u(t)$ which produces the maximum response y_d [Eq. (9)] $u(t)$ will typically be parameterized using a discrete set $a_1, a_2, \ldots, a_n, \ldots$ of real coefficients which define the coordinates of the function space within which the constraint, Eq. (10), is imposed and the multidimensional search performed.

In practice, it will not be feasible to perform an exhaustive search of this multidimensional space, owing to the number of coefficients a_i involved. Therefore, some form of directed search will be required. However, a consequence of system nonlinearity is that (unlike the situation for linear systems) systematic "hill-climbing" procedures can converge to a local, rather than the required global, maximum. Not only must the user satisfy himself that this problem has been overcome, but he will be required to satisfy the certification authority that this is so. Options open to the user include comparing results from searches which start from several different sets of initial conditions. A more comprehensive technique is to combine systematic hill-climbing with the incorporation of a degree of randomization, to produce a stochastic-search method (simulated annealing[9]) that has been widely used in the fields of statistical physics, image processing, and artificial intelligence.

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Coupled Deflection and Rotation of Anisotropic Open-Section Composite Stiffeners

Lawrence C. Bank* and Emmanuel Cofie†
Catholic University of America, Washington, D.C. 20064

Introduction

THIN-WALLED open-section composite material stiffeners of the *T*, *I*, and *J* geometries are routinely used in stiffened-panel construction for structural components in aerospace vehicles.[1] In certain applications, the flange and web elements of the stiffener may need to be constructed of anisotropic panels of composite materials.[2] An analytical method is outlined in this note to perform preliminary hand-calculation analysis of open-section composite stiffeners (beams) that are composed of flanges and webs having anisotropic lay-ups. The predictions of the proposed one-dimensional analytical model are compared with finite-element results using the NISA II composites finite-element code. Analytical beam-type theories for thin-walled composite beams have concentrated on modeling closed cross-sections for rotor blade applications.[3] Beam finite elements have been developed from theories of this type. The analysis of thin-walled open-section anisotropic stiffeners using finite-element analysis, has typically been performed by several authors using specialized beam elements, often based on the Vlasov beam theory.[2,4,5–8] Significant effort has been devoted to the problem of buckling and postbuckling of thin-walled laminated composite structures.[9] Recently Bank[10] and Bank and Cofie[11,12] proposed a one-dimensional theory based on a modified classical beam theory for the linear analysis of generally anisotropic open-section composite beams. The analytical theory, which permits hand-calculation of beam deformations, is intended for preliminary design studies. The theory allows the structural designer to obtain a physical feel for the behavior of anisotropic composite beams which should aid in the intuitive understanding of the behavior of composite structures.

In the theory, open-section stiffeners constructed of laminated panels of composite materials are considered. The panels can be layed-up to have in-plane anisotropy or may be in-plane orthotropic or quasi-isotropic. Each panel of the cross-section may have different mechanical properties, such that a *T* stiffener with an anisotropic flange and an orthotropic web[2] can be analyzed. The analytical theory and calculation method are described in detail elsewhere.[10–12] The fundamental assumptions of classical beam theory are modified to account for anisotropic coupling effects in the proposed analytical theory. As is commonly assumed[13] in the analysis of composite beam structures, only the in-plane stiffness of the panels is assumed to contribute to the overall beam behavior. The cases of pure bending and transverse loading are considered separately. The transverse deflection calculation is different for the two cases since the effect of shear deflection is included for transverse loading but not for pure bending. The procedure for finding the out-of-plane displacement, is however, the same for the two cases.

Modified Beam Theory

Pure Bending

Pure bending of a composite beam subjected to couples at its ends applied in the plane of symmetry of the cross-section is considered. Each panel of the beam is considered to be subjected to a linear stress distribution. Due to the anisotropically induced shear strain in the web and flange panels of the thin-walled section, the beam will deform both in-plane (transverse deflection) and out-of-plane (rotation and lateral deflection), in addition to its conventional transverse deflection related to the axial (bending) strain. If the beam is composed of panels having different mechanical properties, the appropriate mechanical constants for the panel must be used. For the calculation of the overall beam deformation a transformed section approach is used.[14,15] The total deflection y_t, of the beam is given as the sum of a bending deflection y_b, and anisotropic deflection y_a. The anisotropic deflection, due to the induced shear strain component in the web of the section, is found together with the bending deflection utilizing a modification of the Timoshenko beam theory.

Transverse Loading

Symmetric bending of a beam subjected to loading transverse to its cross-sectional plane is considered. The loads may

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*Associate Professor, Department of Civil Engineering. Member AIAA.

†Graduate Student, Department of Civil Engineering. Student Member AIAA.